NAG Toolbox for MATLAB

g01em

1 Purpose

g01em returns the probability associated with the lower tail of the distribution of the Studentized range statistic, via the function name.

2 Syntax

```
[result, ifail] = g01em(q, v, ir)
```

3 Description

The externally Studentized range, q, for a sample, $x_1, x_2, \ldots, x_r$, is defined as:

$$q = \frac{\max x_i - \min x_i}{\hat{\sigma}_e},$$

where $\hat{\sigma}_e$ is an independent estimate of the standard error of the x_i's. The most common use of this statistic is in the testing of means from a balanced design. In this case for a set of group means, $\bar{T}_1, \bar{T}_2, \ldots, \bar{T}_r$, the Studentized range statistic is defined to be the difference between the largest and smallest means, $\bar{T}_{\text{largest}}$ and $\bar{T}_{\text{smallest}}$, divided by the square root of the mean-square experimental error, MS_{error}, over the number of observations in each group, n, i.e.,

$$q = \frac{\bar{T}_{\text{largest}} - \bar{T}_{\text{smallest}}}{\sqrt{MS_{\text{error}}/n}}.$$

The Studentized range statistic can be used as part of a multiple comparisons procedure such as the Newman–Keuls procedure or Duncan's multiple range test (see Montgomery 1984 and Winer 1970).

For a Studentized range statistic the probability integral, $P(q; v, r)$, for v degrees of freedom and r groups can be written as:

$$P(q; v, r) = C \int_0^\infty x^{v-1} e^{-vx^2/2} \left\{ r \int_{-\infty}^{\infty} \phi(y)[\Phi(y) - \Phi(y - qx)]^{r-1}\, dy \right\} dx,$$

where

$$C = \frac{v^{v/2}}{\Gamma(v/2)2^{v/2-1}}, \qquad \phi(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2} \qquad \text{and} \qquad \Phi(y) = \int_{-\infty}^{y} \phi(t)\, dt.$$

The above two-dimensional integral is evaluated using d01da with the upper and lower limits computed to give stated accuracy (see Section 7).

If the degrees of freedom v are greater than 2000 the probability integral can be approximated by its asymptotic form:

$$P(q; r) = r \int_{-\infty}^{\infty} \phi(y)[\Phi(y) - \Phi(y - q)]^{r-1}\, dy.$$

This integral is evaluated using d01am.

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Lund R E and Lund J R 1983 Algorithm AS 190: probabilities and upper quartiles for the studentized range *Appl. Statist.* **32 (2)** 204–210

Montgomery D C 1984 *Design and Analysis of Experiments* Wiley

Winer B J 1970 *Statistical Principles in Experimental Design* McGraw–Hill

5 Parameters

5.1 Compulsory Input Parameters

1: **q – double scalar**

q, the Studentized range statistic.

Constraint: $\mathbf{q} > 0.0$.

2: **v – double scalar**

v, the number of degrees of freedom for the experimental error.

Constraint: $\mathbf{v} \geq 1.0$.

3: **ir – int32 scalar**

r, the number of groups.

Constraint: $\mathbf{ir} \geq 2$.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Note: g01em may return useful information for one or more of the following detected errors or warnings.

ifail $= 1$

On entry, $\mathbf{q} \leq 0.0$,
or $\mathbf{v} < 1.0$,
or $\mathbf{ir} < 2$.

ifail $= 2$

There is some doubt as to whether full accuracy has been achieved.

7 Accuracy

The returned value will have absolute accuracy to at least four decimal places (usually five), unless **ifail** $= 2$. When **ifail** $= 2$ it is usual that the returned value will be a good estimate of the true value.

8 Further Comments

None.

9 Example

```
q = 4.6543;
v = 10;
ir = int32(5);
[result, ifail] = g01em(q, v, ir)
```

```
result =
    0.9500
ifail =
           0
```